

**The difference of the data, the mean and the sample standard deviation is calculated in each example for your convenience. You have to decide in each case which test (paired or two sample) should be applied. So dont be mislead: even if the difference of the paired data is given, it is still possible that the two sample t-test is the correct analysis.**

1. An experiment is conducted on the effect of alcohol on perceptual motor ability. Ten subjects are each tested twice, once after having two drinks and once after having two glasses of water. The two tests were on two different days to give the alcohol a chance to wear off. Half of the subjects were given alcohol first and half were given water first. The scores of the 10 subjects are shown below. The first number for each subject is their performance in the "water" condition. Higher scores reflect better performance. Test to see if alcohol had a significant effect. Use 0.01 significance level!

	Water	Alcohol	Difference
	16	13	3
	15	13	2
	11	10	1
	20	18	2
	19	17	2
	14	11	3
	13	10	3
	15	15	0
	14	11	3
	16	16	0
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Average	15.3	13.4	1.9
Sample std. dev.	2.67	2.95	1.20

2. The scores on a vocabulary test of a group of 20 year olds and a group of 60 year olds are shown below. Test the mean difference for significance using the 0.05 level.

	20 yr olds	60 yr olds	Difference
	27	26	1
	26	29	-3
	21	29	-8
	24	29	-5
	15	27	-12
	18	16	2
	17	20	-3
	12	27	-15
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Average	20	25.38	-5.38
Sample variance	29.14	23.13	35.70

3. Two different analytical tests can be used to determine the impurity level in steel alloys. Eight specimens are tested using both procedures, and the results are shown in the following tabulation.

Specimen	Test 1	Test 2	Difference
1	1.4	1.2	0.2
2	1.7	1.3	0.4
3	1.5	1.5	0
4	1.3	1.4	-0.1
5	2	1.7	0.3
6	2.1	1.8	0.3
7	1.7	1.4	0.3
8	1.6	1.3	0.3
Average	1.66	1.45	0.21
Sample std. dev.	0.28	0.21	0.17

Are the two analytical methods biased to each other? Use 0.05 significance level.

4. Two different analytical tests can be used to determine the impurity level in steel alloys. An alloy is tested using both procedures with eight repetitions and the results are shown in the following tabulation.

Repetition	Test 1	Test 2	Difference
1	1.4	1.2	0.2
2	1.7	1.3	0.4
3	1.5	1.5	0
4	1.3	1.4	-0.1
5	2	1.7	0.3
6	2.1	1.8	0.3
7	1.7	1.4	0.3
8	1.6	1.3	0.3
Average	1.66	1.45	0.21
Sample std. dev.	0.28	0.21	0.17

Are the two analytical methods biased to each other? Use 0.05 significance level!

5. The overall distance traveled by a golf ball is tested by hitting the ball with Iron Byron, a mechanical golfer with a swing that is said to emulate the legendary champion, Byron Nelson. Ten randomly selected balls of two different brands are tested and the overall distance measured. The data follow:

Brand 1: 275, 286, 287, 271, 283, 271, 279, 275, 263, 267 ( $s^2=64.46$ ,  $avg=275.7$ )

Brand 2: 258, 244, 260, 265, 273, 281, 271, 270, 263, 268 ( $s^2=100.90$ ,  $avg=265.3$ )

Test the hypothesis that both brands of ball have equal mean overall distance. Use alpha equals to 0.05.

6. An article in *Neurology* (1998, Vol. 50, pp. 1246–1252) discussed that monozygotic twins share numerous physical, psychological, and pathological traits. The investigators measured an intelligence score of 10 pairs of twins, and the data are as follows:

Pair	Birth Order: 1	Birth Order: 2	Difference
1	6.08	5.73	0.35
2	6.22	5.8	0.42
3	7.99	8.42	-0.43
4	7.44	6.84	0.6
5	6.48	6.43	0.05
6	7.99	8.76	-0.77
7	6.32	6.32	0
8	7.6	7.62	-0.02
9	6.03	6.59	-0.56
10	7.52	7.67	-0.15
Average	6.03	6.59	-0.1
Sample std. dev.	7.52	7.67	0.44

Is there any evidence that mean score depends on birth order? Use alpha equals to 0.05.

### Answers

The answers below help to check the results of the calculation. The correct solution of each problem should include the answers to the practical questions as well. E.g. in example 1 we can say, that the alcohol has a statistically significant effect on perceptual motor ability. Or: we reject the hypothesis that the alcohol does not affects the perceptual motor ability.

1. paired t-test,  $t_0=5.018$ ,  $\pm t_{\alpha/2}=\pm 3.169$ ; the null hypothesis, that the expected values are equal is rejected
2. two-sample t-test,  $t_0=-2.102$ ,  $\pm t_{\alpha/2}=\pm 2.145$ , the null hypothesis, that the expected values are equal is accepted;  $F_0=1.26$ ,  $F_{0.025}=4.99$ , the null hypothesis, that the variances are equal is accepted
3. paired t-test,  $t_0=3.48$ ,  $\pm t_{\alpha/2}=\pm 2.364$ ; the null hypothesis, that the expected values are equal is rejected
4. two-sample t-test,  $t_0=1.736$ ,  $\pm t_{\alpha/2}=\pm 2.145$ , the null hypothesis, that the expected values are equal is accepted;  $F_0=1.79$ ,  $F_{0.025}=4.99$ , the null hypothesis, that the variances are equal is accepted
5. two-sample t-test,  $t_0=2.557$ ,  $\pm t_{\alpha/2}=\pm 2.1$ , the null hypothesis, that the expected values are equal is rejected;  $F_0=1.565$ ,  $F_{0.025}=4.02$ , the null hypothesis, that the variances are equal is accepted
6. paired t-test,  $t_0=-0.366$ ,  $\pm t_{\alpha/2}=\pm 2.262$ , the null hypothesis, that the expected values are equal is accepted